9/5 (목)

S.M black hole ~ 4 million $M_\odot$

1. Celestial Mechanics

(Estimation)

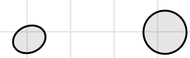
- How many stars (in the Milky way)? ~ 100 – 400 billion
- Binary Stellar System ~ 1/3 – 1/2 of all stars
  - Gravitationally bounded

→ How the motions of individual stars (and masses) can be inferred?

(mass is very important factor in astrophysics

- Newton's 2nd Law : $\vec{F} = m\vec{a}$
  - Simple case : m_1, m_2 where $m_1 \gg m_2$
  - m is the mass of stellar object (Neutron stars, white dwarf, blackholes ...)

1) Binary star systems



Brightness
↔ distance,
(= Astrometry)

- Visual binaries (ex: Kruger 60)
- Astrometric binaries (ex: AB Doradus) → 하나의 별이 흔들리는 모습 관측.

Optical
- Visual binaries
- Eclipsing binaries ($i \sim 90°$)
- Spectroscopic binaries

+ astrometric binaries

Mostly born together during evolution.

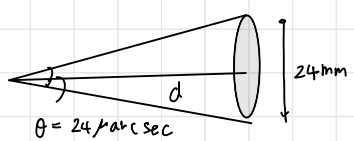
→ very close
[공전 주기)
[line-of-sight velocity.

→ V_r ↑, LT. 대부분의 경우 $d <$ 1AU, 식도 같이 발생 (Eclipsing spectroscopic)

- Parallax $d = \dfrac{1AU}{\tan p}$

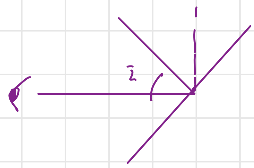
1PC = 206265 AU = 3.086×10^{16} m = 3.26 lyr

ex)



$d \sim 2 \times 10^8$ m

(cf: $\pi^2 = 10$ not 9. ◡̈)



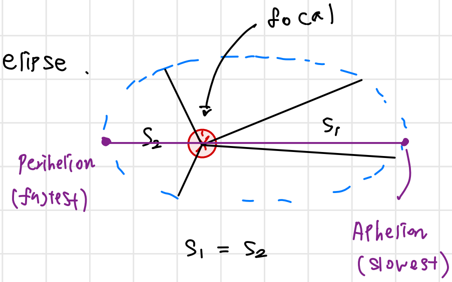
24mm 가 d 만큼 떨어지면 24×10^{-6} arc sec @

$$\Rightarrow 24 \times 10^{-6} \text{ arc sec} = \frac{24\text{mm}}{d} \Rightarrow d = \frac{24 \times 10^{-3} \text{ m}}{24 \times 10^{-6} \text{ arc sec} \times \dfrac{\frac{\pi}{648000} \text{ rad}}{1 \text{ arc sec}}}.$$

9/10 (화) : Basic Mechanics

1. Kepler's Laws

1) The orbital track of the planets follows the elipse.



2) The radius vector (sun → planet) sweeps out equal areas in equal time ⟺ 각운동량 보존.

3) $G M p^2 = 4\pi^2 a^3 \quad (M \gg m)$

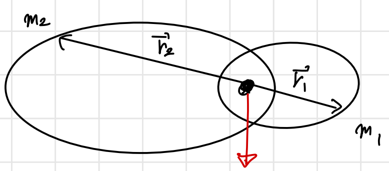
p ↳ orbital period; a ↳ semimajor axis length

⇒ Equation of elipse

$$r(\theta) = \frac{a(1-e^2)}{1 + e\cos\theta} \quad (0 \le e \le 1)$$

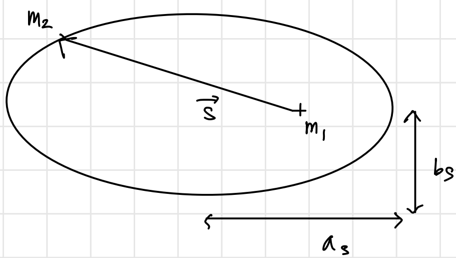
$$\frac{b}{a} = \sqrt{1-e^2}$$

2. Two-body Problem : where is the reference frame?



$$\vec{S} = \vec{r}_2 - \vec{r}_1$$

(Assumption: Not accelerating B.C. due to external force)



⟨m_1 reference frame⟩

$$\vec{B} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

$= 0$ ($\because$ Barycenter at the origin, $\vec{B} = 0$)

$\therefore m_1\vec{r}_1 + m_2\vec{r}_2 = 0.$

⇒

$$\vec{r}_1 = -\frac{m_2}{m_1 + m_2}\vec{S} = -\frac{\mu}{m_1}\vec{S}$$

$$\vec{r}_2 = \frac{m_1}{m_1 + m_2}\vec{S} = \frac{\mu}{m_2}\vec{S}$$

$\mu \equiv \dfrac{m_1 m_2}{m_1 + m_2}$ (Reduced mass), $M_T = m_1 + m_2$

3. Equation of motion

$$\vec{F}_{21} = m_2 \frac{d^2\vec{r}_2}{dt^2} \qquad \vec{F}_{12} = m_1 \frac{d^2\vec{r}_1}{dt^2}$$

$\vec{F}_{12} + \vec{F}_{21} = 0$ (Newton's 3rd law)

$$\Rightarrow \frac{\vec{F}_{21}}{m_2} - \frac{\vec{F}_{12}}{m_1} = \frac{d^2}{dt^2}\underbrace{(\vec{r}_2 - \vec{r}_1)}_{=\vec{S}} \Rightarrow -\frac{G m_1 m_2}{S^3}\vec{S} = \mu\frac{d^2\vec{S}}{dt^2}, \quad \therefore -\frac{G M_T \mu}{S^3}\vec{S} = \mu\frac{d^2\vec{S}}{dt^2}$$

Equivalent 1-body equation

4. Solutions of the equation of motion.

- Angular momentum (Kepler's 2nd law) $J = m_1 r_1^2 \omega + m_2 r_2^2 \omega \Rightarrow \mu S^2 \omega$

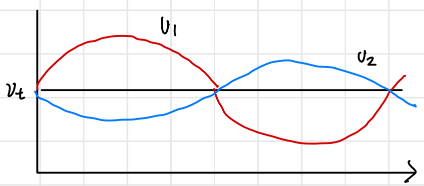
$$J = \sqrt{\frac{G M_T}{a_S}}\,\mu b_S$$

- Elliptical Motion $s(\theta) = \dfrac{J^2}{G M_T \mu^2} \cdot \dfrac{1}{1 + e\cos\theta}$

- Kepler III. $G M_T P^2 = 4\pi^2 a_s^3$
- Total energy $E_t = -\dfrac{G M_T \mu}{2 a_s}$

Try to show this (My homework)

5. Mass Determinations : Radial velocities in a spectroscopic Binary system



Ideal Graph of radial velocity

- Spectroscopic binary (Doppler shift)

$$G P^2 \frac{m_2^3}{(m_1 + m_2)^2} = 4\pi^2 a_1^3 \quad \left(a_1 = \frac{m_2}{m_1 + m_2} a_s \right)$$

$$\boxed{\frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2}} = \frac{4\pi^2}{G P^2} (a_1 \sin i)^3$$

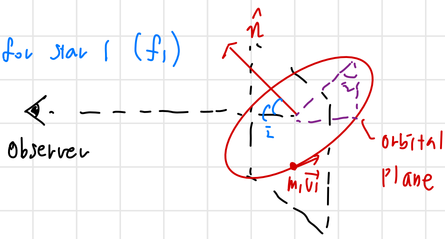
mass function for star 1 (f_1)

Q. Why we need i (inclination)?

→ The line of sight velocity

$v_r(t) = \underline{(v_1 \sin i)} \sin \omega t$

↳ Maximum value of v_r



$$f_1 = \frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2} = \frac{P}{2\pi G} (v_1 \sin i)^3 \xrightarrow[m_1 \ll m_2]{} f_1 \sim m_2 \sin^3 i \qquad \left(P = \frac{2\pi a_1}{v_1} \right)$$

cf) 같은 맥락

blackhole
(Stellar mass
(Super mass

9/12 (목) : Celestial Mechanics + History of Dark matter

Galileo's Discoveries → Invisible things become visible with tech-development

"Dark"? → What we don't know.

1) Dark stars : Black hole! John Michell's imagination

2) Dark planet :
- Neptune
- Vulcan (?) → Einstein's t.o.R.

3) Milky way structure

(*) Evidence of Dark matter

1. Galaxy cluster

Zwicky's galaxy cluster mass determination : Mt. wilson telescope. (caltech)

비리얼 정리 $\langle v \rangle \sim 80 km/s$ vs $200 km/s$

invisible mass ? → indirect evidence of D.M.

2. Galaxy.

- Not atom
- 관측 X
- 강한 약력 전자기에도 상호작용X
- 차가운 물질

9/19 (목) 2. Equilibrium in Stars.

· solid angle $\Omega = \frac{A}{r^2}$. $d\Omega = \sin\theta \, d\theta \, d\varphi$.

· flux $F = \frac{L}{4\pi r^2}$.

* Sun (Stellar) Energy Sources : " What is the process that produce the star's Luminosity?"

1) Chemical Energy

$$\begin{cases} L_\odot = 4\times10^{26} W \\ M_\odot = 2\times10^{30} kg \\ R_\odot = 7\times10^{8} m \\ d = 1AU = 1.5\times10^{11} m \end{cases}$$

Age ~ 4.5 billion yrs.

18th ~ : " Burning" - Oxidation.

· Let's assume if every atom in the Sun is available to release ~ 10eV.

How long it will take for the sun to use all ability?

$\Rightarrow n = \frac{M_\odot}{m_H} = \frac{2\times10^{30} kg}{1.6\times10^{-27} kg} \sim 10^{57}$ atoms $\longrightarrow \frac{E}{L}$ 로 t 구하면 (lifetime)

$t \sim 10^5$ years

Too short!!

2) Gravitational Energy : Shinning ∵ g potential

⇒ Sun should be large before (Shrinking happens)

· "How long the Sun could shine if grav potential E → only Source?"

$$\Rightarrow du = -\frac{GM_r}{r}dm = -\frac{GM_r \cdot (4\pi r^2)\rho}{r}dr \quad \text{where} \quad \langle\rho\rangle = \frac{M_r}{\frac{4\pi}{3}r^3}$$

$$\Rightarrow u \sim 4\pi G \langle\rho\rangle^2 \int_0^R \frac{4\pi}{3} r^4 \, dr \sim \frac{3GM^2}{5R} \sim \frac{GM^2}{R}.$$

$\xrightarrow[\text{Virial thm}]{} \sim 10^{41} J$. ∴ $t \sim 10^7$ Years, Still short.

$\Delta E \sim -\frac{1}{2}u = \frac{3GM^2}{10R}$

cf) Accretion

부착 (강착)

3) Nuclear Energy : $E = mc^2$

· Atomic mass unit : $u = 1.660540\times10^{-27}$ kg.

H : $m_H = 1.007825\, u$

He : $m_{He} = 4.002603\, u$

$|m_{He} - 4\cdot m_H| = 0.028677\, u$.

(~ 0.7% m_{He})

$\Rightarrow \Delta E = 26.7\,MeV.$ $= \dfrac{\Delta E}{4M_H} = 6.4\times10^{14}\,J/kg.$

$E \sim \underline{(0.1 M_\odot)} \times \overline{0.007 \times c^2} \sim 10^{44}\,J.$ $t \sim 10^{10}\,yr.$ ※ point.

Assumption ?

"Stability" → key to describe stellar interior

- Size ? : Size requirement in star formation
→ Balance between E_K and E_P
- Time scales
- Luminosity

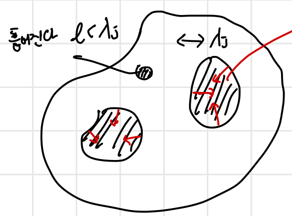
→ Different system ex) Cluster

· Stability

- Size requirement in star formation
- Hydrostatic Equilibrium : Virial theorem
- Mass, Luminosity

2.2. Jeans Length : 성간 구름이 분리하려면, 퍼텐셜이 운동 에너지 보다 커야한다!

①



⟨Gas Cloud⟩

Perturbation of size $> \lambda_J$ ⇒ Continue to Contract

· Gravitational Collapse Condition $E_K \lesssim |E_P|$

$\Rightarrow \underbrace{\left(\frac{1}{2}m_H v^2\right)_{av}}_{=\text{ Average } E_K} \lesssim \dfrac{GMm_H}{R}$ (M = mass of the cloud)

$\boxed{M \sim \rho R^3}$

$\Rightarrow \boxed{v^2 \le G R^2 \rho}$ ... Critical Size for collapse $R \gtrsim \boxed{\dfrac{v}{\sqrt{G\rho}}}$

(order ~1 factor 무시)

⇒ Jeans length

$\equiv \lambda_J = \dfrac{v_s}{\sqrt{G\rho}}$

$v_s^2 = \dfrac{\gamma P}{\rho} = \gamma \dfrac{kT}{m}$ $(\gamma \to \frac{5}{3})$

(γ = ratio of specific heat)

where v_s is the speed of sound (ordering!!!)

② Critical Mass $M_c \sim \rho \cdot \lambda_J^3 = \dfrac{v_s^3}{G^{\frac{3}{2}}\,\rho^{\frac{1}{2}}} \approx \boxed{\left(\dfrac{kT}{Gm}\right)^{\frac{3}{2}} \dfrac{1}{\sqrt{\rho}}}$ $v_s \sim \sqrt{\dfrac{kT}{m}}$

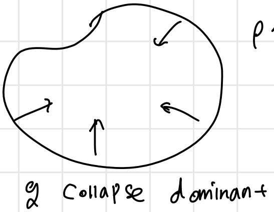
= Jeans mass

(※) Example.

· Neutral Hydrogen cloud of ISM (Interstellar medium)

· Number density (typical) : $n = 4\times10^7\,m^{-3}$ $T \sim 100K$ (Lower density compared to our daily life)

(After calculation) $\begin{cases} M_c \sim 3000 M_\odot \\ \lambda_J \sim 60\,lyrs \end{cases}$



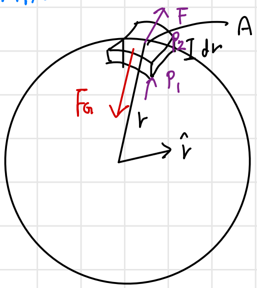
$P \uparrow$

- Assume $\times \frac{1}{100}$ size
- $T \sim 100K$

$\Big] \; M_c' \sim \frac{1}{10^3} M_c$

$\Rightarrow$ From one giant group, single, hundreds ~ thousands of stars formulating.

2.3. Hydrostatic Equilibrium : No net force



$\vec{F_1} = P_1 A \hat{r} \quad \vec{F_2} = -P_2 A \hat{r}$. Each mass element has no net force on it.

$P_1, P_2 \rightarrow$ Gas pressure

$dP = P_2 - P_1$ (upward) < 0

$F_p = -A\,dP$

$\Downarrow \; -A(P_2 - P_1)$

$F_G + F_p = 0.$

$= -\frac{1}{r^2} G M(r) \rho(r) A\,dr$

$$\Rightarrow \frac{dP}{dr} = -\frac{G M(r) \rho(r)}{r^2} = -\rho(r)\, g(r) \qquad g(r) = \frac{GM(r)}{r^2}$$

(Adr 로 나눠줌)

$M(r)$: r 내부 총 질량

$\rho(r)$: r에서의 밀도.

2.4. Virial Theorem : 내부 구조 고려 X, 에너지로 평형상태 설명하는 도구 (암흑 물질 !!)

$$V(r)\,dr = \frac{4}{3}\pi r^3 dr. \qquad V(r)\,dP = -\frac{1}{3} \frac{GM(r)}{r} \underbrace{[4\pi r^2 \rho\, dr]}_{= dM}$$

(Valid for $V \sim \frac{1}{r}$)

$$\Rightarrow \int_{star} V(r)\,dP = \underbrace{\int_{star} -\frac{GM(r)}{r}\, dM \cdot \frac{1}{3}}_{= \frac{1}{3} \sum E_p}$$

(Sum of Potential energy of stars)

where $d(PV) = PdV + VdP$ → 적분.

$\begin{cases} V \rightarrow 0 & r = 0 \\ P \rightarrow 0 & r = R \end{cases}$

$$\int_{star} PdV + \int_{star} VdP = 0$$

$$\int VdP = -\int PdV$$

$$\Rightarrow -3\int PdV = \sum E_p$$

(Statistical mech arguement)

$(P = nkT)$

For a perfect, non-relativistic gas, $P = \frac{2}{3} n \left(\frac{1}{2} m v^2\right)$ · plug this in above.

$\int \frac{2}{3} u_K\, dV$

↳ kinetic energy density

$$\Rightarrow \boxed{2\sum E_K + \sum E_p = 0, \quad E_{tot} = \sum E_K + \sum E_p = \frac{1}{2}\sum E_p = -\sum E_K}$$

… Virial thm

→ Good relation of understanding ideal condition physical property.

→ 별의 비밀 : ex) 핵이 수축하면 오히려 온도 상승. → 은하단의 안정성 : 암흑물질.

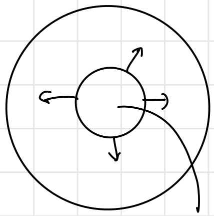
ex) Zwicky's galaxy cluster mass determination

G.C. : Relatively stable assumption-able → $\underline{\text{virial Mass}}$ → Existence of D.M.

$$M = \frac{2\langle v_z^2\rangle_{av}}{G\langle r_{ij}^{-1}\rangle_{av}}$$

2.6. Nuclear burning : 음성피드백 메커니즘

- Power Source



· Sun : relatively - stable stage period $\sim 10^{10}$ years

- Multiple - chain reaction involved.
- P-P (Proton) reaction → Ionized condition
- H, He, Metal.

Q. How to maintain stable equilibrium?

- Star is perturbed to a small size $R\downarrow$.

→ $P, T \uparrow$

- Increased Energy released. (More Nuclear reaction)
- $R\uparrow$ (Increase the size)

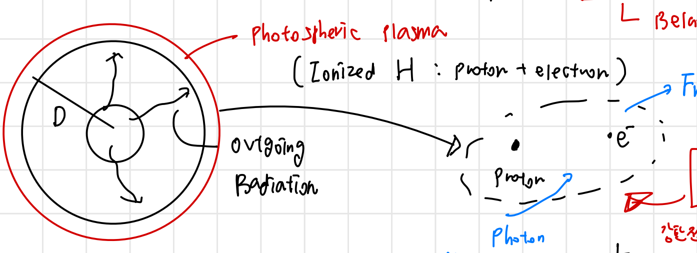
(Inverse direction is possible)

⇒ Self-regulating, Negative - feedback

2.7. Eddington Luminosity

- Practical maximum to the luminosity of $\underline{\text{(normal)}}$ star

└ Relatively stable, hydrostatic



Force?

1) Grav → Proton mass >> electron mass

양성자에게 강한 힘 작용

2) Radiation pressure

강한 전기력에 → Compton scattering (전자에 작용)

인해 크게 분리 X, 하나의 단위로 끌려가는

energy flux $[W/m^2] = \frac{L}{4\pi D^2}$

1) $F_{rad,e} = \underbrace{P_{rad}}\cdot\underbrace{\sigma_T}_{=\text{톰슨 산란 단면적}} = \left(\frac{F}{c}\right)\sigma_T$

$= \frac{L\sigma_T}{4\pi D^2 c}$

2) $F_G = -\frac{GM\mu_e m_p}{D^2}$ (electron molecular weight)

i) fully ionized H: $\mu_e = 1$

ii) ″ He: $\mu_e = 2$

$\left(\because P = \frac{E}{c},\ = (\text{Force})(\text{time}),\ \text{pressure} = \frac{\text{Force}}{\text{Area}} = \frac{\text{Force}\cdot\text{time}}{\text{Area}}\right)$

⇒ $F_{rad,e} + F_G = 0.$

⇒ $$\boxed{L = \frac{4\pi G M_\odot \mu_e m_p c}{\sigma_T}\left(\frac{M}{M_\odot}\right)[W] = 1.26\times10^{31}\mu_e\left(\frac{M}{M_\odot}\right)}$$

... Eddington Luminosity

if $L > L_{Edd}$: Expansion, not-stable ∴ L_{Edd} is maximum point with stability.

→ significant mass loss happens.

For sun, $L = 10 x L_\odot$ → Very stable.

실제로 $L \propto M^{16/5}$. but $L \propto M$. → 어느 순간 L 이 L_{edd} 따라잡아,

그 이상 질량 ($M \sim 130 M_\odot$) 별은 관측 X.

3. Stellar Structure and Evolution

⟨Stellar Structure⟩ → 구형 대칭 / 회전 X 가정

Fundamentals
1) Hydrostatic Equilibrium
2) Mass Distribution
3) Luminosity Distribution
4) Energy transport

+ Equation of state $P(\rho, T)$

Stellar Model

1) HSE : $\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2} = -g(r)\rho(r)$ (Net force = 0)

2) Mass Distribution (Spherical Model)

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

3) Luminosity

$L(r)$: Power passing through spherical surface of radius r

· ε [W/kg] → 에너지 생성함수 (단위 질량 당 전력 생성) → $\varepsilon(r,T) = \varepsilon(r)$

· $\varepsilon_{pp} = \varepsilon_0 X^2 \left(\frac{\rho}{10^5\ \mathrm{kg\,m^{-3}}}\right)\left(\frac{T}{10^7\ \mathrm{K}}\right)^{\beta}$ [W/kg] ... Matter content, density, temp function.

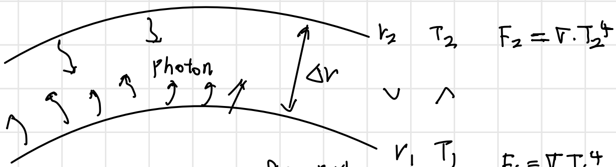
→ 2.5×10^{-3} W/kg $(= \varepsilon_0 ?)$

4) Energy transport.

- Radiation(복사)

or

- Convection (대류)



Δr = mean free path

$= \frac{1}{\kappa \rho}$

↳ Opacity (cross-section per mass) [m^2/kg]

⇒ Assume perfect black body.

$F = \sigma T^4$.

$\Rightarrow F_{net} = \sigma (T_1^4 - T_2^4)$ $[W/m^2]$ where $F = \frac{L}{4\pi r^2}$.

$\Rightarrow L_{rad}(r) = -4\pi r^2 (\sigma T_2^4 - \sigma T_1^4) = -4\pi r^2 \frac{d}{dr}(\sigma T^4)\Delta r = -4\pi r^2 \cdot 4\sigma T^3 \cdot \frac{dT}{dr} \cdot \frac{1}{\kappa \rho}$

$T(r)$

$\frac{4}{3}$배, why?

$$\boxed{\frac{dT(r)}{dr} = -\frac{3}{64\pi\sigma}\frac{\kappa(r)\rho(r)}{T^3(r)r^2}L(r)}$$ ... Radiation

Otherwise, for the case of Convection, (상승, 단열 팽창, 냉각 → 주변보다 여전히 뜨거움 → 계속 상승)

$$\boxed{\left(\frac{dT(r)}{dr}\right)_{adiabatic} = -\left(1-\frac{1}{\gamma}\right)\frac{T}{P}\cdot\frac{GM(r)\rho(r)}{r^2}}$$ ... Convection

$\left|\frac{dT}{dr}\right|_{rad} > \left|\frac{dT}{dr}\right|_{adiabatic}$

- Sun

$R < 0.7 R_\odot$: Radiation

$R > 0.7 R_\odot$: Convection ⟶ Nuclear burning is happening in core.

$\Rightarrow$

4.3 Stellar Model

4 fundamental equations

\+

Secondary equations ⟶ Equation of states $P = \frac{\rho}{m_{av}} kT$

cf) Saha eq.

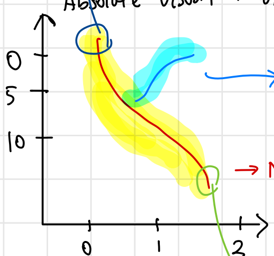
cf) Exoplanet finding

1. Radial velocity
2. Transit
3. Direct Imaging
4. Gravitational Microlensing
5. Astrometry

< HR diagram : A plot of $\log L - \log T$ >

High L, hot stars (most) massive

Absolute visual magnitude (same d), Luminosity ↑



→ Giant branch. Why? ⇒ Evolution of stars!

→ Main sequence (H-burning stable)

B-V (mag)

blue filter

Visual (Yellow)

Lower luminosity, cool stars, least massive

temp ↑

- Luminosity (at the outer surface) of a star

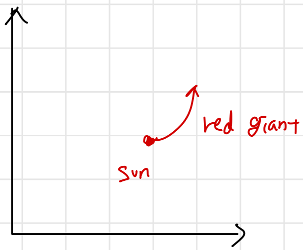
$L = 4\pi R^2 \cdot \sigma T_{eff}^4$

Radius ↵ (R) ; ↳ effective temperature (T_{eff})

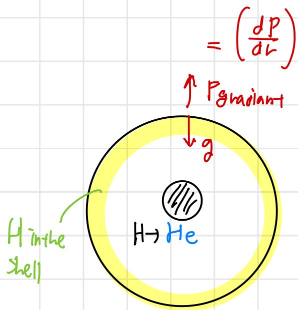
ex) For Sun,
$T_{eff} \sim 5800K$
(cf. $T_{eff} < T$)

- Solar Evolution

- Stars form from the ISM gases → "Proto Stars"
- The Star settles on MS. (Function of Mass)
  → Zero-age MS



HR of sun

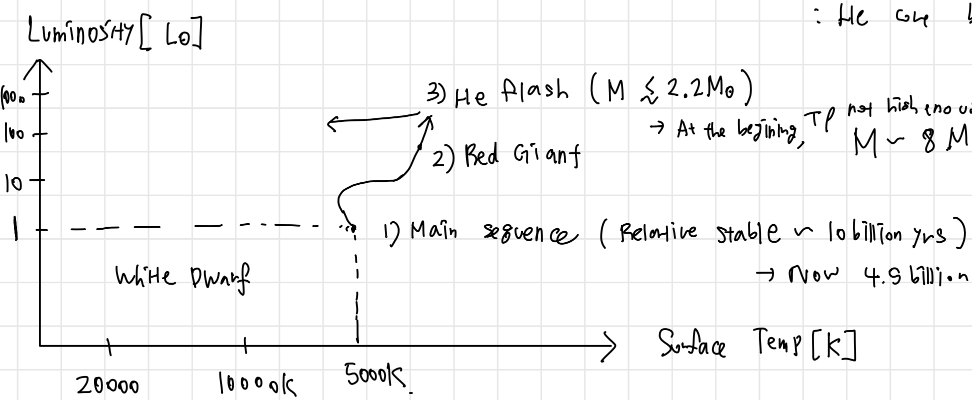


- When H is exhausted, what happens?

The core shrinks,
⇒ $R\downarrow$, $T, \rho \uparrow$
H burning (outside) increased burning rate
⇒ He burning
(Red Giant)
- $R \Uparrow$, $L\uparrow$, $T\downarrow$
⟶ C, O, Ne (Heavy elements)

10/15 Evolution of Single Star ($M_\odot$)

- $M \lesssim 2.2 M_\odot$
  : He core becomes degenerate
  Same Star ↵



→ At the begining, T_p not high enough.
$M \sim 8 M_\odot$

→ H burning
→ Now 4.5 billion yrs

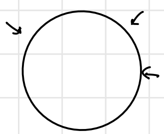
⟨ Solar mass star ⟩

- H burning (at the core) ends
  → Evolution off the main sequence, (He core and residual H shell)
  → Core begins to contract, while H around the core burning still.
  → T rises in the shell. → $L \uparrow \xrightarrow[\text{but}]{} T_{eff} \downarrow$. ($\because R \Uparrow$)

- Red Giant
  → Expansion of the envelope & $T_{eff} \downarrow$.
  → H^- ion forms. → Convection (the efficiency of E transport ↑)
  → More E transport, expansion!!

- He flash (Positive feedback)
  → Collapsing He core
  → the core : electron - degenerate $\quad P \sim \rho k T \xrightarrow{\text{high temp}} P \propto \rho^{5/3}$ (Not a function of Temp anymore)
  → $T \sim 10^8 K$, $\rho \sim 10^7 \, kg\,m^{-3}$, He burning

If $P(\rho, T)$ → Stability. But, if $P(\rho)$, think about little contract perturbation.


He core

$T\uparrow, \rho\uparrow. \xrightarrow{\vec{g} \text{ Collapse}} T \Uparrow \rho \Uparrow$, not enough.
Also He burning↑.

$F = \frac{dP}{dr}$ pressure gradiant for He burning.

$M \leq 8 M_\odot$
$M \geq 8 M_\odot$ Heavier elements

4.4. Compact Stars : The final state of a star (WD, NS, BH)

- EOS $P(\rho, T) \Rightarrow$ $$P_e = \frac{1}{20}\left(\frac{3}{\pi}\right)^{\frac{2}{3}} \frac{h^2}{m_e}\left(\frac{\rho}{\mu_e m_p}\right)^{\frac{5}{3}} \quad \cdots \text{Non-relativistic}$$
  $M \gtrsim 1.4 M_\odot$
  $P_e \propto \rho^{\frac{5}{3}}$ $$= \frac{1}{8}\left(\frac{3}{\pi}\right)^{\frac{1}{3}} ch \left(\frac{\rho}{\mu_e m_p}\right)^{\frac{4}{3}} \quad \cdots \text{relativistic}$$

- $M \gtrsim 1.4 M_\odot$: 축퇴압 ↓, WD 붕괴.

(기억 필요X)

White Dwarf (Supported by Degenerate of electron) : Chandrasechra Mass limit
→ Max mass of a stable WD

- $$M_{limit} = \frac{\omega_3 \sqrt{3\pi}}{2}\left(\frac{\hbar c}{G}\right)^{\frac{3}{2}} \frac{1}{(\mu_e m_H)^2}$$

$$\frac{R}{R_\odot} \simeq 0.01 \left(\frac{M}{M_\odot}\right)^{-\frac{1}{3}} \Rightarrow \downarrow R \propto \frac{1}{\uparrow M^{\frac{1}{3}}} \text{ !!}$$

Blackhole $\begin{cases} \text{Stellar} & \sim 10 M_\odot \\ \text{Supermassive} & \sim 10^6 - 10^8 M_\odot \end{cases}$

(intermediate

→ very little is known

· $R_S = \frac{2GM}{c^2}$. $(M = M_\odot \rightarrow R_S \sim 3\,km)$

M, J, Q
$\hookrightarrow 0$
(Astrophysical scale)

· Angular momentum

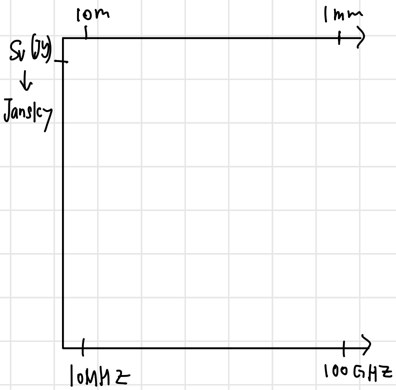
$$R_h = \frac{GM}{c^2}\left[1 + \left(1 - \left(\frac{J}{J_{max}}\right)\right]^{\frac{1}{2}}\right], \quad J_{max} = Mc \cdot \frac{GM}{c^2} \Rightarrow ISCO$$

ISCO : "stable" orbit with circular motion closest to the blackhole.

CH4.

Radio (4MHz, 7m wavelength)

< The nature of radio sources >



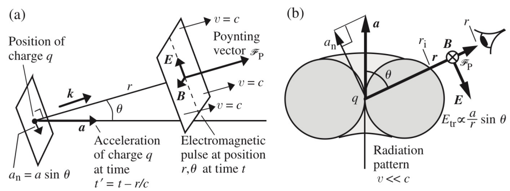


Fig. 5.2: (a) Electric and magnetic field vectors, ***E*** and ***B***, at the position r, θ at time t that arise from the horizontal acceleration ***a*** of a positive charge at the earlier time, $t' = t - r/c$. The two planes shown are normal to the propagation vector $\boldsymbol{k} = (2\pi/\lambda)\,\hat{k}$, where the hat indicates a unit vector. The ***E*** and ***B*** fields are constant over the entire right-hand plane. (b) Dipole radiation pattern for a vertical acceleration ***a*** of a positive charge. The radial distance r_i along the line of sight from the origin to the intercept of the circle represents the relative magnitude of the transverse electric vector. At a fixed observer distance r, the magnitude varies as $\sin\theta$; see (4). The radiated power is $\propto E^2$. The radiated ***B*** vector points into the paper.

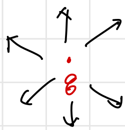
< Bremsstrahlung : Free electron Radiation, + optically thin >

→ 광학적으로 얇은, 비상대론적 plasma의 thermal 제동 복사

⇒ J_ν (ν, T) 유도하자 (복사 방출률)

(*) Review of E&M

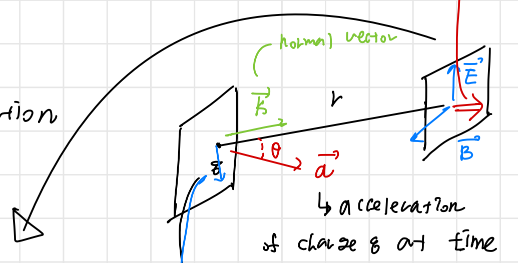
· Radiated electric vector



$\vec{E}$: Instantaneous pattern of E field of a charge q

⇒ moving in a straight line at speed $v \ll c$

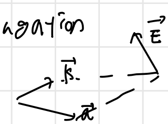
· If it is non-relativistic charge ⇒ Acceleration



· The propagating $\vec{E}$ due to a previous instantaneous acceleration

→ lies in the plane defined by the $\vec{a}$ & the propagation vector $\vec{k}$

EM pulse at position r, θ at time t



· For a positive charge, $\vec{E}$ is directed opposite to and has amplitude proportional to the projection a_n (of $\vec{a}$) on the plane normal to the line-of-sight

- Its amplitude is proportional to charge q, and varies with distance as $\frac{1}{r}$.
  + the energy flux is proportional to $E^2 \sim \frac{1}{r^2}$

⇒ The magnitude of the radiated transverse E field (in a vacuum)

$$\therefore E_{tr} \propto \frac{q a_n}{r} = \left(\frac{qa}{r}\right)\sin\theta.$$

t' 가속도

$$\vec{E}(r,t) = E_{tr}\,\hat{n} = \frac{q\,a(t')\sin\theta}{4\pi\varepsilon_0 c^2 r}\,\hat{n} = \frac{q}{4\pi\varepsilon_0 c^2 r}(\vec{a}\times\hat{k})\times\hat{k}$$

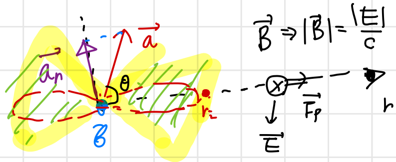
($v \ll c$ valid)

SI unit

$\hat{n}$: unit vector in the transverse direction normal to $\vec{k}$ in the plane of $\vec{E}$ & $\vec{a}$

"Dipole Radiation pattern"

⇒ for a vertical acceleration $\vec{a}$ of a positive charge:



$\vec{B} \Rightarrow |\vec{B}| = \frac{|\vec{E}|}{c}$

$E_{tr} \propto \frac{a\sin\theta}{r}$ (transverse E field)

r observer

The power radiated $\propto E^2$.

$\hat{k}(\vec{k})$: $\vec{E}\times\vec{B}$

- The distance r_i from the origin to the intercept of the l-o-s with the outer boundary of the doughnut-shaped pattern

⇒ Relative magnitude of the field at the angle (θ) for a fixed observation distance r

(*) Poynting vector

- At any given point (at time t), the energy flux density $\vec{F}$ (W/m^2) carried by an EM wave depends only on:
  the instantaneous values of $\vec{E}$, $\vec{B}$ of the wave at that position (and time, speed of propagation)

- From the energy densities [J/m^3] of E & B fields

$\varepsilon_0 E^2/2$ ↵ ↳ $B^2/2\mu_0$ + $B = E/c$

$$\Rightarrow \vec{F}_p = \frac{\vec{E}\times\vec{B}}{\mu_0} \ (W/m^2)$$

$\mu_0 = 4\pi\times10^{-7}$ (T./m)

$\varepsilon_0 = 8.854\times10^{-12}$ (F/m)

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

+ Because $\vec{E} \perp \vec{B}$, $|\vec{F_p}| = \varepsilon_0 c\, \vec{E} \cdot \vec{E} = \varepsilon_0 c E_{tr}^2$

$$\boxed{|\vec{F_p}(r,\theta,t)| = \frac{q^2 \sin^2\theta\, a^2(t')}{(4\pi)^2 \varepsilon_0 c^3 r^2} \quad [W/m^2]}$$

→ 핵심은, (r,θ,t) 에서의 $|\vec{F_p}|$ 가 θ, t'의 함수이다.

"Larmor's Formula"

- The total power radiated by the electron into all direction

→ Summation over a spherical surface at distance r.

$$\Rightarrow \boxed{P(t) = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} F_p(r,\theta,t)\, r^2 \sin\theta\, d\phi\, d\theta = \frac{1}{6\pi\varepsilon_0} \cdot \frac{q^2 a^2(t)}{c^3} \quad [W]}$$

t' → t로 치환. 순간적인 방출 power 로 해석.

∴ Instantaneous emitted power

→ t'에서의 가속에 의한, t 에서 r 구면 통과하는 전체 power

→ No r dependence

11/5

(ex) Unit of radiation

- Intensity (brightness) $[W\,m^{-2}\,sr^{-1}]$
- Specific Intensity (spectral intensity) $[W\,m^{-2}\,sr^{-1}\,Hz^{-1}]$
- Flux $[W\,m^{-2}]$
- Flux density $[W\,m^{-2}\,Hz^{-1}]$
- Luminosity $[W]$

dependence on Area

Q. 결국 무엇으로 측정하나?

Voltage !!

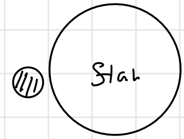
→ Multi-wavelength astronomy.

m^{-2} [area] — detection (distance of detector)

Hz^{-1} $[\nu, \lambda]$ — wavelength dependence (frequency)

Sr^{-1} [Solid angle] — Source size

cf)



⟨ Bremsstrahlung ⟩

→ $K \gg 13.6\,eV$, 완전히 이온화 된 원자으로 알려져 있음.

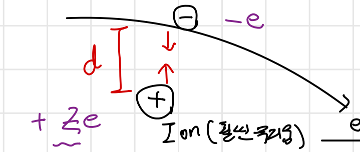
· Intracluster Medium (ICM) ($10^5 \sim 10^6 K$) radiation.

→ Ionized gas radiation. (plasma) → 이상적으로는 연속 스펙트럼

· Free-Free radiation — Coulomb collision between electrons & ions

(Hot, ionized gas) = Hot plasma

└ Equilibrium ⇒ Maxwell - Boltzmann distribution



electron → 정지 상태로 간주

(Z=1) : Hydrogen

decelerated :⇒ radiation

$$\frac{3}{2} k_B T = \left\langle \frac{1}{2} m v^2 \right\rangle$$ mass and speed of individual atoms.

define T

광학 깊이 $\tau \ll 1$

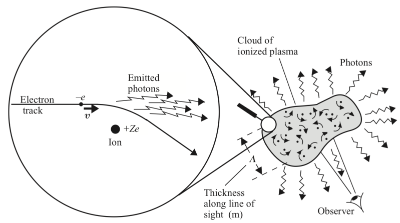


Fig. 5.1: Cloud of plasma (ionized gas) giving rise to photons owing to the near collisions of the electrons and ions. The electrons are accelerated and thus emit radiation in the form of photons. The line-of-sight thickness of the cloud is Λ.

· For $E_k \sim 13.6\,eV$, $T \rightarrow 10^5 K$.

Assumptions.

Ion is Stationary ($M \sim 2000 m_e$)

electron is moving not very fast ($v \ll c$) : Non relativistic case

⇒ 고전 전자기가 커버 가능.

· Classical, "Semi-quantitative" derivation ($K \gg h\nu$)

(다음 양들을 차례로 구함)

1) Radiative E by single electron

2) Frequency (ν) of the emitted radiation

(~ electron speed, impact parameter b) 사이의 관계

3) Power emitted at ν, in $[\nu, \nu+d\nu]$ applying M-B distribution for velectrons

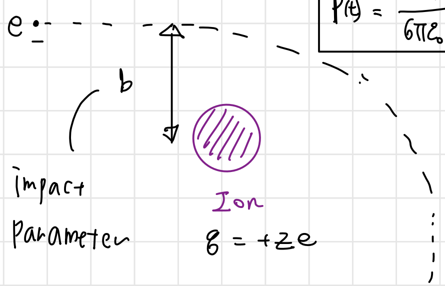
⇒ Volume emissivity $j_\nu(\nu, T) \xrightarrow{\int j_\nu d\nu} J(T) \rightarrow I(\nu, T)$ specific Intensity.

$W m^{-3} Hz^{-1}$

↓ 1m³ 부피 내 모든 충돌 고려

↓ 주파수 적분

↓ Line of Sight 두께에 따른 Specific intensity



$$\boxed{P(t) = \frac{1}{6\pi\varepsilon_0}\frac{q^2 a^2}{c^3}}$$ → Larmor formula

i) Energy radiated per collision

- $\vec{a} = \frac{\vec{F}}{m} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}\frac{1}{m}\hat{r}$ → +e 방향

$$|\vec{a}|_{max} \simeq \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{mb^2}$$

- Time interval during which the acceleration is compatible to a_{max} (for v)

"Collision time" $\tau_b \simeq \frac{b}{v}$

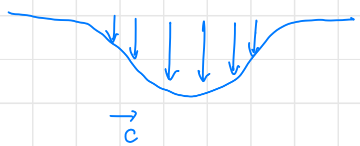
⇒ Total E (충돌 동안 방출된)

$$Q(b,v) = \int_{-\infty}^{\infty} P(t)\,dt$$

$$= \frac{1}{6\pi\varepsilon_0}\cdot\frac{e^2}{c^3}\int_{-\infty}^{\infty} a(t)^2\,dt \simeq \frac{1}{6\pi\varepsilon_0}\frac{e^2}{c^3}a_{max}^2\,\tau_b = \frac{1}{(4\pi\varepsilon_0)^3}\frac{2}{3}\cdot\frac{Z^2e^6}{c^3m^2b^3v}$$

→ b가 감소할때 따라 Q 급격히 증가 $b\downarrow$, $Q\Uparrow$

→ $v\uparrow$, $Q\downarrow$



ii) Frequency of the emitted radiation

Single pulse of $\vec{E}$ (Rough Estimation)

$$\omega \approx \frac{1}{\tau_b} \approx \frac{v}{b}$$

⇒ Characteristic frequency $\nu = \frac{\omega}{2\pi} \simeq \frac{v}{2\pi b}$

⇒ $b = \frac{v}{2\pi\nu}$ ⇒ $db = -\frac{v}{2\pi\nu^2}d\nu$

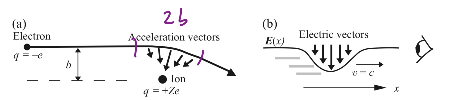
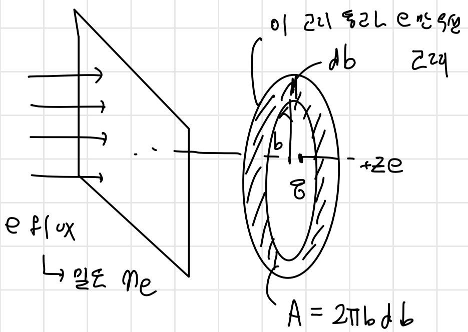


Fig. 5.3: Electron trajectory showing (a) deviation of track, impact parameter b, and acceleration vectors and (b) the pulse of emitted transverse electric vectors for a negative charge. The profile $E(x)$ at a fixed time is shown as a solid line.

5.4. 'thermal' electrons & a single ion.



(*) The power emitted by electrons as a function of (ν, v)

electron # / s

$$P_b(b,v)\,db = Q(b,v)\,n_e v\,2\pi b\,db$$

Power coming from each ion

Electron flux [electrons $m^{-2}s^{-1}$]

단위 시간당 방출 에너지

$P_b(b,v) \to P_\nu(\nu,v)$

(단위 db 당) → (단위 dν 당)

$$\int_{b_1}^{b_2} P_b(b,v)\,db = -\int_{\nu_1}^{\nu_2} P_\nu(\nu,v)\,d\nu$$ [W]

$$\therefore \; P_\nu(\nu, v) = -P_b(b, v)\frac{db}{d\nu} = \overset{\text{대입}}{Q(b,v)\, n_e v \cdot 2\pi \bar{b}}\, \frac{v}{2\pi\nu^2}$$

$$\simeq \left[\frac{1}{(4\pi\varepsilon_0)^3}\cdot\frac{8\pi^2}{3}\, n_e\, \frac{Z^2 e^6}{c^3 m^2 v}\right] d\nu \;(\mathrm{W/ion}) \quad \rightarrow \nu \text{ 의존 X.}$$

b ↑ 가속 ↑ (힘 ↑) ⇒ ν↑, 수↓ ⇒ 균형!

b↑ 가속 ↓ (힘 ↓) ν↓ 수↑

(*) Electrons of many speeds

- For a non-degenerate gas

$$P(v)\,dv = \underbrace{P(\vec{v})}_{\text{M-B distribution of particle speeds}} \cdot 4\pi v^2\, dv$$

$$P(\vec{v}) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \cdot \exp\left(-\frac{mv^2}{2k_B T}\right)$$

$$\langle P_\nu(\nu)\rangle_{\mathrm{ion}} = \int_{v_{min}}^{\infty} P_\nu(\nu, v)\, P(\vec{v})\, 4\pi v^2\, dv \quad [\mathrm{W \cdot ion^{-1} \cdot Hz^{-1}}]$$

→ 전자 바다 속 단일 이온이 주파수 ν에서 방출하는 power

$$v_{min} = \sqrt{\frac{2h\nu}{m}}$$ → 양자적 제한

(iv) Spectrum of emitted photons 단위부피당 단위 Hz에서

· Volume emissivity $j_\nu(\nu)$ [$\mathrm{W\, m^{-3}\, Hz^{-1}}$] → 모든 방향!

of radiation

$$j_\nu(\nu)\,d\nu = \underbrace{n_i}_{\substack{\text{ion density}\\ [\mathrm{ion \cdot m^{-3}}]}}\; \underbrace{\langle P_\nu(\nu)\rangle_{\mathrm{ion}}}_{\text{단일 이온}}\; \underbrace{d\nu}_{\text{진동수}} = n_i\left[\int_{v_{min}}^{\infty} P_\nu(\nu, v)\, P(\vec{v})\, 4\pi v^2\, dv\right] d\nu \quad [\mathrm{W\,m^{-3}}]$$

$$= \underbrace{g(\nu, T, Z)}_{\text{Gaunt factor}}\, \frac{1}{(4\pi\varepsilon_0)^3}\cdot\frac{32}{3}\left(\frac{2}{3}\frac{\pi^3}{m^3 k_B}\right)^{\frac{1}{2}} \cdot \frac{Z^2 e^6}{c^3}\, n_e n_i\, \frac{e^{-h\nu/k_BT}}{T^{\frac{1}{2}}}\, d\nu$$

$$= \underbrace{C_1}_{6.8\times10^{-51}\ \mathrm{J\cdot m^3\cdot K^{1/2}}}\, g(\nu, T, Z)\, Z^2 n_e n_i\, \frac{e^{-h\nu/k_BT}}{T^{1/2}}\, d\nu$$

g : slowly varying factor

⇒ why?

①

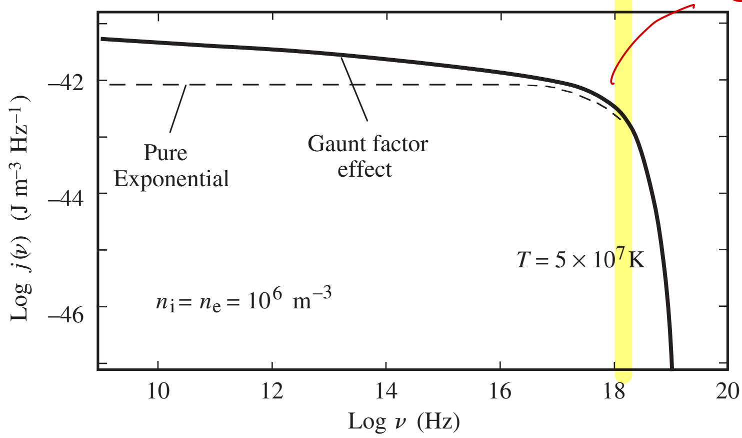
② $e^-\ e^-\ e^-$ → ion screening effect

e^- . .

Take into account.

Pretty flat, not totally flat

sharp cut off



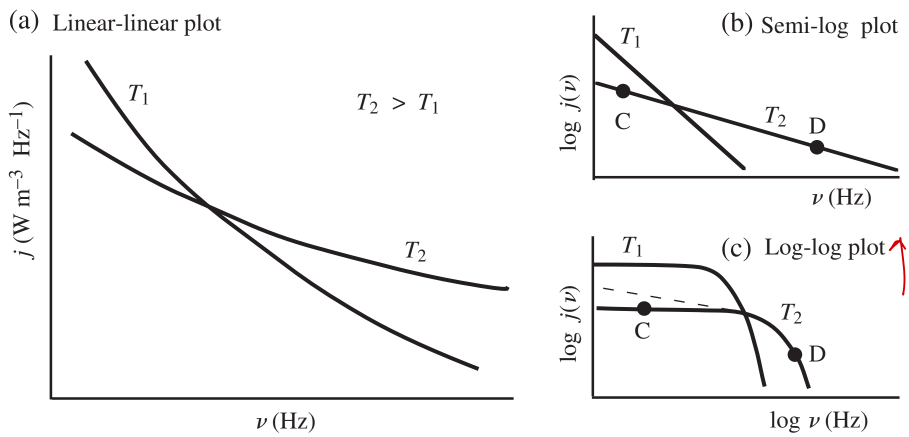
$\frac{h\nu}{kT} \sim 1.$

10^{44}

$\frac{10^{18} \cdot 6.6\times10^{-34}}{1.3\times10^{-23}} \sim T.$

$5\times10^{7}.$

Fig. 5.5: Theoretical continuum thermal bremsstrahlung spectrum. The volume emissivity (37) is plotted from radio to x-ray frequencies on a log-log plot with the Gaunt factor (38) included. The specific intensity $I(\nu, T)$ would have the same form. Note the gradual rise toward low frequencies due to the Gaunt factor. We assume a hydrogen plasma ($Z = 1$) of temperature $T = 5 \times 10^7$ K with number densities $n_i = n_e = 10^6$ m^{-3}.



$\nu = \frac{v}{b}$

(eventually captured, not anymore free-free interaction

Fig. 5.6: Thermal bremsstrahlung spectra (as pure exponentials) on linear-linear, semilog, and log-log plots for two sources with the same ion and electron densities but differing temperatures, $T_2 > T_1$. Measurement of the specific intensities at two frequencies (e.g., at C and D) permits one to solve for the temperature T of the plasma as well as for the emission measure $\langle n_e^2 \rangle_{av} \Lambda$. [From H. Bradt, *Astronomy Methods*, Cambridge, 2004, Fig. 11.3, with permission]

$\exp(-h\nu/kT) \approx 1.0$. The dashed curve in Fig. 5.5 is thus flat as it extends to low frequencies. The effect of the Gaunt factor is shown; it modifies the exponential response noticeably but modestly over the many decades of frequency displayed.

The curves in Fig. 5.6 qualitatively show the function $j_\nu(\nu, T)$ on linear, semilog and log-log axes for two temperatures $T_2 > T_1$. The exponential term causes a rapid reduction ("cutoff") of flux at a higher frequency for T_2 than for T_1. At low frequencies, because the exponential is essentially fixed at unity, the intensity is governed by the $T^{-1/2}$ term if

$$j(T) = \int_0^\infty j_\nu(\nu)\,d\nu = C_2\,\bar{g}(T,Z)\,Z^2 n_e n_i T^{1/2} \quad [W/m^{-3}]$$

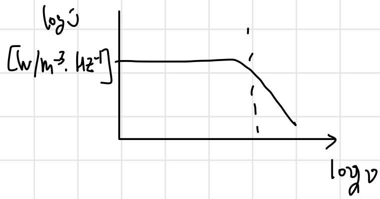
$\boxed{}$ 1.44×10^{-40} $\cancel{W/m^3}$ $K^{-1/2}$

Wm^3.

Luminosity $L(T) = \int_{\text{volume of sphere}} j(T)\,dV \quad [W]$

$\uparrow$ $\nu = \frac{v}{b\downarrow}$ (eventually captured, not anymore free-free interaction

Galaxy cluster → spectral radiation



Hydrogen plasma

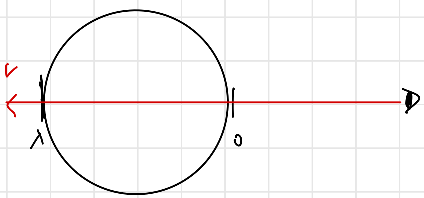
$$L(T) = \int j(T)\,dV \propto \int_V n_e^2\,dV \quad (n_e \sim n_i)$$

(*) specific intensity $I_\nu(\nu,T)$ $[W m^{-2}\ Hz^{-1}\ Sr^{-1}]$

If the emission is isotropic, $I_\nu(\nu,T) = \int_0^\lambda \frac{j_\nu(r,\nu,T)}{4\pi}\,dr$ (radial position along line of sight

$$= \frac{C_1}{4\pi}\,g(\nu,T)\,\frac{e^{-h\nu/k_BT}}{T^{1/2}} \cdot \underline{\int_0^\lambda n_e^2\,dr} \quad [W m^{-2}\ Hz^{-1}\ Sr^{-1}]$$

= emission measure $[m^{-5}]$



(*) (Spectral) flux density

$$S(\nu,T) = \iint I(\nu,T)\,d\Omega \quad [W m^{-2}\ Hz^{-1}]$$

$$= \frac{1}{4\pi r^2}\cdot j_{\nu,av}(\nu,T)\,4\pi R^3 \cdot \frac{1}{3}$$

Optically thin ⇒ Scatter X ???

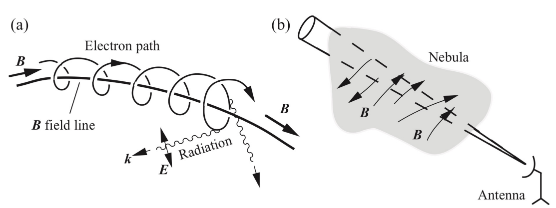


Fig. 8.1: (a) Electron spiraling around a magnetic field line emitting polarized radiation in the electron's forward direction. (b) Line of sight through a nebula with partially ordered magnetic fields.

5. Syncrotron radiation

$E > 10^{11}$ eV. How? ⇒ Crab Pulsar

Crab nebula → 편광된 빛, "beam"

polarized emission → Syncrotron { 주파수 ?, 파워 P, 특성시간 (전자의 E_K 앎) ⊏

관측
편광

(*) Syncrotron radiation (Magnetic bremsstrahlung)

— "Relativistic" electron spiraling around B fields

구경 ∝ 파장.

VLA → Radio

Chandra → X-ray

$\vec{F} = q(\vec{v} \times \vec{B})$

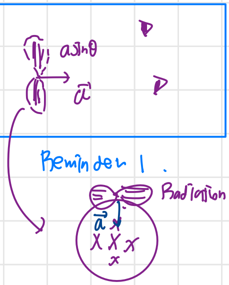
⇒ Charge acceleration → Photon radiation E_K ↓

{ Bremsstrahlung (E)
{ Syncrotron (B)

↳ Non thermal radiation (∵ $E_K \sim T$ ↑)

Larmor's formula $P = \frac{1}{6\pi\varepsilon_0} \cdot \frac{q^2 a^2}{c^3}$ [W] $(v \ll c)$

→ 가속 전하 방출하는 전자기 복사의 power (비상대론적인 경우)



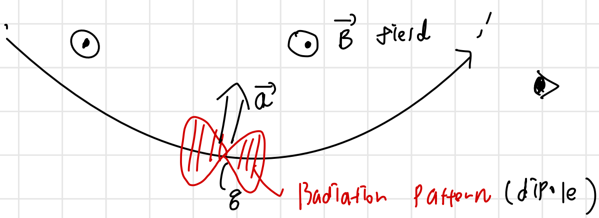
(*) Instantaneous Radiation Patterns

→ 수정이 필요

→ 실제 상대론적 경우 훨씬 크다

a) Classical case $(v \ll c)$

$E_{tr}(t) \propto \frac{q\,a(t')}{r} \sin\theta$



관찰

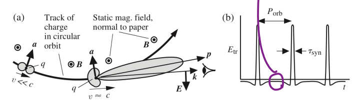


Fig. 8.4: (a) Radiation patterns for a positive charge moving with speed v in a circle in the presence of a uniform magnetic field $\boldsymbol{B}$ directed out of the paper. The instantaneous acceleration is toward the center of the circle. For $v \ll c$, the radiation pattern is the toroid shown in Fig. 5.2b. For $v \approx c$, the radiation is peaked strongly in the forward direction. (b) Plot of the transverse component of $\boldsymbol{E}$ versus time detected by the observer in (a) for a single charge with $v \approx c$. The dominant frequencies detected are approximately equal to the inverse of the duration τ_{syn} of each pulse. Note the finite negative field between pulses and the sudden field reversals at each edge of a pulse. The width of the relativistic forward lobe in (a) and that of the individual pulses in (b) are greatly exaggerated. [After V. Ginzburg and S. Syrovatskii, *ARAA* **3**, 297 (1965)]

가정 { 원궤도 위치 — linear polarization
{ 원운동 — Circular polarization

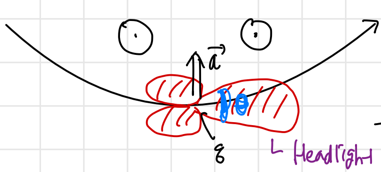
⇒ "cyclotron frequency" $\omega_B = \frac{qB}{m}$ [rad s^{-1}]

b) Relativistic $(v \sim c)$

궤도 평면상 관측자

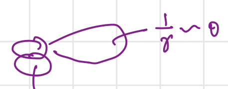
Lorentz transformation $\vec{E}$ & $\vec{B}$ fields.

$\beta = \frac{v}{c} \rightarrow 1$ (rel)

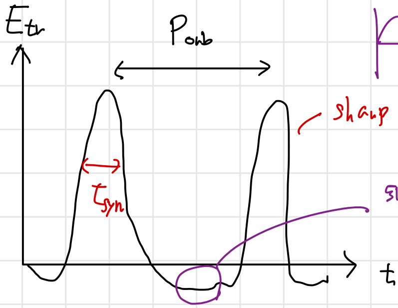


→ Narrowly beamed shape radiation

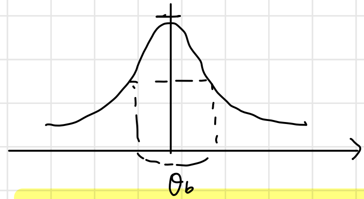
└ Headlight beam



$\nu_- = \nu_0(1-\beta) \rightarrow 0.$
$= \frac{1}{2\gamma^2}$ ...



• Angular half-width of the beam



$$\theta_b \approx \frac{mc^2}{u} = \frac{1}{\gamma} \quad [\text{rad}]$$

total E of the charge

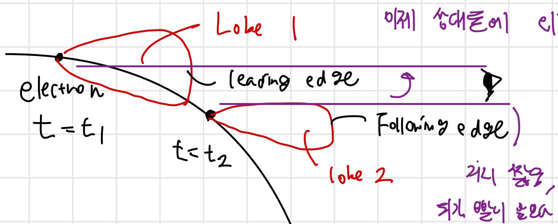
$$= \frac{1}{\sqrt{1-\beta^2}}$$

classical 에서 $\omega_B = \frac{qB}{m}$

$\xrightarrow{\text{relativistic case}}$ $\omega = \frac{1}{\gamma}\omega_B = \frac{1}{\gamma}\frac{qB}{m}$

$\Rightarrow \tau_0 \sim \frac{2\theta_b}{\omega} = \frac{2m}{qB}$ → no γ dependence!

이제 상대론에 의한 Doppler Compression 고려



τ_0 (Full width of the beam to pass over the observer)

$$\sim \frac{2\theta_b}{\omega} = \frac{2}{\omega_b}.$$

→ $\vec{E}$ pulse travels at infinite speed from q to the observer. (일 때 해당한 듯하다)

→ Photons have finite speed.

− the emitting charge : $v \sim c$

Doppler Compression of the pulse of radiation.

시간 간격 $(1-\beta)$ 만큼 축소 (상대론)

$\Rightarrow \tau_0 \downarrow$

$$\boxed{\tau_{syn} = \tau_0(1-\beta) \simeq \frac{\tau_0}{2\gamma^2} = \frac{1}{\omega_b}\frac{1}{\gamma^2}}$$

$$(1-\beta) = \frac{1-\beta^2}{1+\beta} \simeq \frac{1}{2\gamma^2} \quad (\beta \sim 1)$$

• Observed frequency

$$\omega_{syn} \approx \frac{1}{\tau_{syn}} = \gamma^2 \cdot \omega_b \quad [\text{rad}\,s^{-1}]$$

Characteristic frequency of Syncrotron radiation

$$\nu_{syn} \simeq \frac{1}{2\pi}\cdot\left(\frac{u}{mc^2}\right)^2 \frac{qB}{m}\sin\varphi \quad [\text{Hz}]$$ (dominant freq)

cf) $E(t) \xrightarrow[\text{변환}]{\text{푸리에}} \tilde{E}(\omega)$

φ : pitch angle : charge 방향 / B 방향 각도.
$\vec{v}$ $\vec{B}$

- ν_{syn} (& u)

i) Low E $\quad \gamma = \dfrac{u}{mc^2} \sim 1 \quad \rightarrow \quad 2\pi \nu_{syn} = \dfrac{qB}{m} \quad (\sin\varphi = 1) \quad \rightarrow$ cyclotron freq.

ii) $\gamma \gg 1$ $\quad$ (ex) $u = 10^{11}\,eV \quad m_e c^2 = 5\times10^5\,eV$ $\quad$ (Crab nebula case?)

$\gamma = 10^5. \quad \Rightarrow \quad \omega_{syn} \sim 4\times10^{10}\,\omega_B$

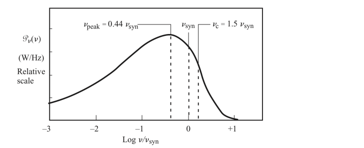
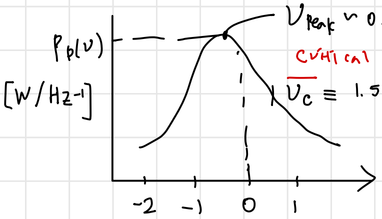


Fig. 8.7: Distribution of radiated power $\mathcal{P}(\nu)$ (W/Hz) plotted with respect to log ν/ν_{syn} for a single orbiting electron. The vertical axis is a relative linear scale. This gives the relative power in the component frequencies of the waveform (Fig. 8.4b) of the radiated electric field. The peak and critical frequencies are indicated with their ratios relative to ν_{syn}. Note that νsyn is a function of the pitch angle ϕ. [After B. Rossi and S. Olbert, *Introduction to the Physics of Space*, McGraw Hill, 1970, p. 42; data from V. Ginzburg and S. Syrovatskii, Table 1, *ARAA* **3** 297 (1965)]

(*) For a single electron : Radiated Power

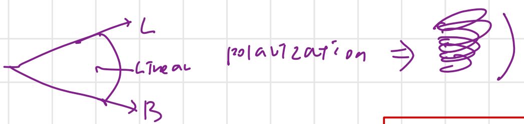


$\nu_{peak} \sim 0.4\,\nu_{syn}$

critical frequency

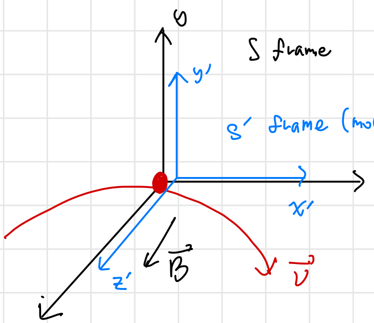
$\nu_c \equiv 1.5\,\nu_{syn} = 6.3\times10^{36}\,u^2 B \sin\varphi$ [J^2][T] (for a single electron)

$\leftarrow$ Calculate $E_{tr}(t)$ & F.T $\rightarrow P_\nu(\nu)$



polarization $\Rightarrow$ Linear

8.4. Power radiated by the electron



$$\frac{du'}{dt'} = -P' = -\frac{1}{6\pi\varepsilon_0}\frac{q^2 a'^2}{c^3}$$

(rate of energy change in S')

where $\left[\begin{array}{l} E_y' = -\gamma\beta c B_z = -\gamma v B_z \\ B_z' = \gamma B_z \end{array}\right.$

정지 → 자기력 0, 전기력만!

$(q = Ze)$

$$a' = \frac{F_y'}{m} = \frac{qE_y'}{m} = -\frac{Ze\gamma v B}{m} \overset{v\sim c}{=} -\frac{ZeBu}{m^2 c}$$

$u = \gamma mc^2$

$$\frac{du'}{dt'} = -\frac{1}{6\pi\varepsilon_0}\frac{q^2a'^2}{c^3} = -\frac{1}{6\pi\varepsilon_0}\frac{(Ze)^2}{c^3}\left(\frac{ZeBu}{m^2c}\right)^2 = -\frac{1}{6\pi\varepsilon_0}\frac{1}{c^6}\left(\frac{Ze}{m}\right)^4 u^2B^2$$

[W]

($p\sim1$, $q\sim T\mu z$)

$\therefore$ Relativistic transformation

$\left.\begin{array}{l} du = \gamma\, du' \\ dt = \gamma\, dt'. \end{array}\right] \quad \underset{(S)}{\dfrac{du}{dt}} = \underset{(S')}{\dfrac{du'}{dt'}}.$

Instantaneous rate of E loss (by the particle)

$$\frac{du}{dt} = -\frac{1}{6\pi\varepsilon_0}\cdot\frac{1}{c^5}\cdot\left(\frac{Ze}{m}\right)^4\cdot u^2 B^2 \beta^2 \sin^2\varphi \quad [W]$$

$$[W] = -2.37\times10^{12}\, u^2 B^2 \beta^2 \sin^2\varphi$$

(전자) $\longrightarrow$ $\left(\frac{Z}{m}\right)$ value 가 큰, $m\downarrow$ 일 때 $\Uparrow$

⇒ "전자"

· Energy density of the B field.

$$u_B = \frac{B^2}{2\mu_0} \quad [J\ m^{-3}] \leftarrow (전자기 에트)$$

where $\langle\sin^2\varphi\rangle_{av} = \frac{1}{4\pi}\int_{sphere}\sin^2\varphi\, \underline{d\Omega} = \frac{2}{3}$

$2\pi\sin\varphi\, d\varphi$

$$\frac{du}{dt} = -\frac{4}{3}\sigma_T c \beta^2\gamma^2 u_B = -2.66\times10^{-20}\beta^2\gamma^2 u_B \quad [W]$$

($\because$ $\sigma_T = \frac{8}{3}\pi r_e^2$, $r_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{mc^2}$, $u = \gamma mc^2$, $c^2 = \frac{1}{\varepsilon_0\mu_0}$ 대입 in $\frac{du}{dt}$, 정리)

↳ (고전적 반지름 of 전자)

$$\tau \sim \frac{u}{du/dt} = \frac{5.16}{B^2}\frac{1}{\gamma}\ [s] \quad (\beta\sim1,\ \varphi\sim\frac{\pi}{2})$$

(전자)

$B\sim10^{-6}$.. 이면 $\tau \Uparrow$.

⇒ enough radiation time.

└ characteristic time scale

8.5. Ensemble of radiating particle

[Spectral volume emissivity J_ν $[W\ m^{-3}\ Hz^{-1}]$

Specific intensity I $[W\ m^{-2}\ Hz^{-1}\ sr^{-1}]$

electron population ⟷ energy u.

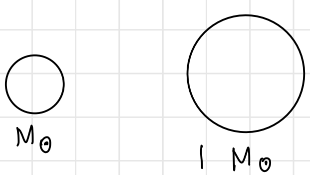
"Power Law"

For independent variable x, $f(x) = a x^{-k}$. $\longrightarrow f(cx) = c^{-k} f(x)$

→ Scale invariant property

→ self - similarity !

ex)



Number specific - intensity [particles $s^{-1} m^{-2} sr^{-1}$] at u in du

$J(u)\,du \propto u^p\,du$. (Definition / assumption)

$(p\sim-2.5)$

- Number density [particles in m^{-3}]

$n(u)du \propto u^p du$

energy specific intensity $I_p(u)$

$I_p(u)du \propto u^{p+1}du$

싱크로트론 일

- For a single electron, $\frac{du}{dt} \propto -u^2B^2$. ($\beta \sim 1$, $\phi \sim \frac{\pi}{2}$)

$j_u(u)du = -\frac{du}{dt}\, n(u)du$ $\sim u^2B^2$ $\sim u^{p+2}\cdot B^2$.

u^p

Photon volume emissivity [$Wm^{-3}J^{-1}$]

$j_u(u) \longrightarrow j_\nu(\nu)$

Synchrotron

$$\frac{du}{dt} = -\frac{1}{6\pi\varepsilon_0}\frac{1}{c^5}\left(\frac{Ze}{m}\right)^4 u^2B^2\beta^2\sin^2\phi \quad [W] \quad \rightarrow \text{Single electron}$$

$$n(u)du = n_0\left(\frac{u}{u_0}\right)^p du \quad [\text{particles } m^{-3}]$$

number density

- Volume emissivity $\quad \frac{du}{dt} \propto -u^2B^2 \quad (\beta \sim 1 \quad \phi = \frac{\pi}{2})$

$$\Rightarrow j_u(u)du = -\frac{du}{dt}n(u)du \propto u^{p+2}B^2du \quad [Wm^{-3}]$$

↳ Photon volume emissivity

$$= \left(\frac{\nu}{B}\right)^{\frac{p+2}{2}}B^2 = B^{\frac{2-p}{2}}\nu^{\frac{2+p}{2}}$$

① ②

$j_\nu(\nu)$ [$Wm^{-3}Hz^{-1}$]

$$j_\nu(\nu)d\nu = j_u(u)du \Rightarrow j_\nu(\nu) = j_u(u)\frac{du}{d\nu}$$

+ charecteristic freq

$\nu_c \propto u^2B$

$\Rightarrow u \propto \sqrt{\frac{\nu}{B}}$

$\Rightarrow \frac{du}{d\nu} \propto B^{-\frac{1}{2}}\nu^{-\frac{1}{2}}$

Combining ① and ②;

$$\therefore j_\nu(\nu)d\nu \propto B^{\frac{-p+1}{2}}\cdot \nu^{\frac{p+1}{2}}d\nu \quad [Wm^{-3}]$$

Power law ($\alpha = \frac{p+1}{2}$)

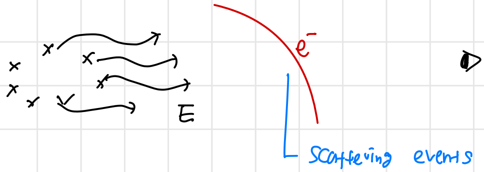
$$\xrightarrow{\text{Normalization}} j_\nu(\nu)d\nu = j_\nu(\nu_0)\left(\frac{B}{B_0}\right)^{-\alpha+1}\left(\frac{\nu}{\nu_0}\right)^{\alpha}d\nu$$

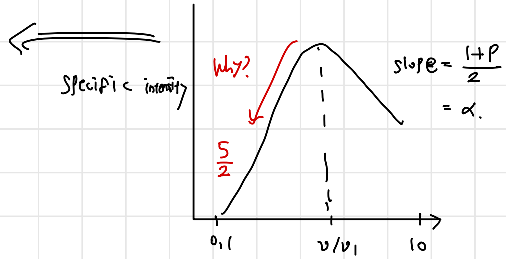
usually $p<-1$, $\alpha<0$

$$j_\nu(\nu) = \frac{e^3}{\varepsilon_0 mc}\,a(p)\,n_0u_0\left(\frac{mc^2}{u_0}\right)^{2\alpha}B^{-\alpha+1}\left(\frac{4\pi m\nu}{3e}\right)^{\alpha} \quad [Wm^{-3}Hz^{-1}] \quad (\beta\sim1)$$

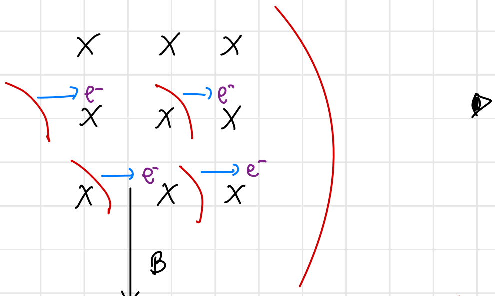
dimless, ~0.1

1)





2)

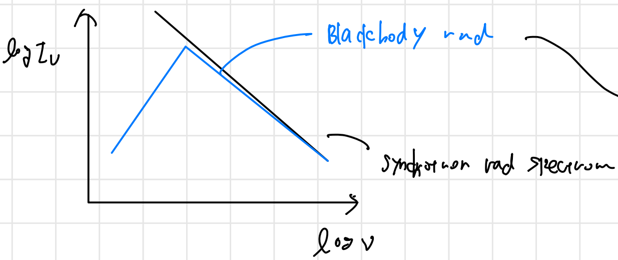


탈출 X

밖에서 관측 X ⇒ partially observe the total flux by synchrotron.

cf) optically thin / thick

cf) physical explanation



↑ absorption cross section (of electron) & low energy photon

→ limit of efficiency (Quasi-thermo equilibrium)

SM Black hole → Synchrotron radiation : main rad

(*) wave length 바꿔가면서 ($\lambda \uparrow \downarrow$) 우리은하 중심 radiation 본다.

1cm → 1mm ($\lambda \downarrow$, $\nu \uparrow$). → 내부 arbitrary rad of SMBH detail animation.

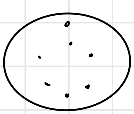
① $\lambda \lesssim 1$mm (synchrotron)

② Angular resolution $\propto \frac{\lambda}{D}$

$\theta \sim 10^{-15}$ μ arcsec 2 μas

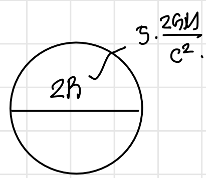
- Event Horizon telescope

③ (physical size) + distance



$D \sim 10000$km $\lambda \lesssim 1$mm

→ $\frac{R}{d}$

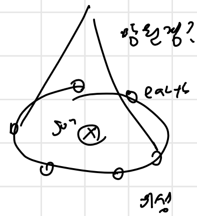


(apparent size) $\propto \frac{M_{BH}}{d}$

Sgr A*
M87* $\sim 40\,\mu as$

distance to the SMBH

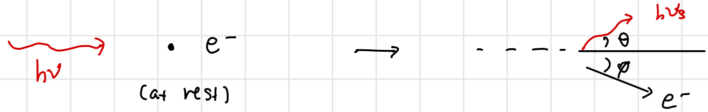
luckily, able to observe!



Compton Scattering

〈 Normal Compton Scattering. 〉

- A photon interacts with a "stationary" electron



- Energy conservation $h\nu + mc^2 = h\nu + \gamma mc^2$
- Momentum Conservation (longitudional) $\frac{h\nu}{c} = \frac{h\nu_s}{c}\cos\theta + \gamma\beta mc\cos\phi$

  (transverse) $0 = \frac{h\nu_s}{c}\sin\theta - \gamma\beta mc\sin\phi$.

$\Rightarrow \cos^2\phi + \sin^2\phi = 1$

$\Rightarrow h^2(\nu^2 + \nu_s^2 - 2\nu\nu_s\cos\theta) = (\gamma\beta mc^2)^2$

$\Rightarrow$ (energy eq$)^2$

$\Rightarrow h^2(\nu^2 - 2\nu\nu_s + \nu_s^2) + 2hmc^2(\nu - \nu_s) + m^2c^4 = (\gamma mc^2)^2$ & $\gamma^2(1-\beta^2) = 1$

$$\boxed{\frac{1}{h\nu_s} - \frac{1}{h\nu} = \frac{1-\cos\theta}{mc^2} \quad \text{or} \quad \lambda_s - \lambda = \frac{h}{mc}(1-\cos\theta)}$$

Photon scattering angle

$\Rightarrow h\nu_s = \dfrac{h\nu}{1 + \frac{h\nu}{mc^2}(1-\cos\theta)}$

λ_c ... 전자 정지 좌표계에서 Compton scattering

= Compton wavelength = 2.43 pm (0.5 MeV)

$\Delta\lambda \to$ 0.5 MeV 해당해야 관측 $\to f \Uparrow \to$ X ray

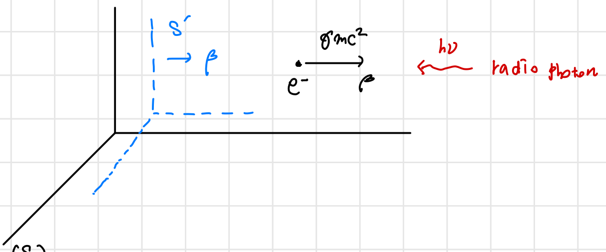
⟨ Inverse Compton Scattering ⟩

- Plasma of relativistic electrons
- electron E > synchrotron photon's E.

1) Transform to the frame of reference (electron is at rest)
2) The collision is then viewed in this rest frame.

$\beta = \frac{c}{v} \sim 1 \quad \theta = \pi$ (반사) → 정면충돌, 최대 에너지

(a) Before collision (S)



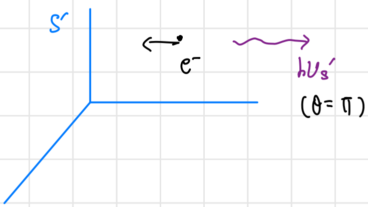
(b) Before collision (S')

$(h\nu' > h\nu)$

↓ 상대론적 도플러

$$h\nu' = \sqrt{\frac{1+\beta}{1-\beta}}\, h\nu$$

(c) After collision (S')

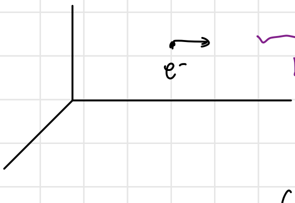


The scattered photon E

$$h\nu_s' = \frac{h\nu'}{1 + \frac{2h\nu'}{mc^2}} \quad \text{(normal Compton scattering)}$$

(d) After collision (S) (두번의 도플러 변환 : E⇑ (→ 방향전환)
산란 : E↓)

$$h\nu_s = \sqrt{\frac{1+\beta}{1-\beta}}\, h\nu_s' \quad \text{(2nd Doppler effect)}$$



$h\nu_s$ (optical / X-ray photons)
↳ Boosted by relativistic electron

for $\beta \sim 1$ $(\gamma \gg 1)$,

$$\sqrt{\frac{1+\beta}{1-\beta}} \sim 2\gamma.$$

$$\therefore h\nu_s \simeq 4\gamma^2 h\nu \left(1 + \frac{4\gamma h\nu}{mc^2}\right)^{-1} \simeq 4\gamma^2 h\nu = 4\left(\frac{u}{mc^2}\right)^2 h\nu$$

→ 상대적으로, E가 γ^2배!

대부분의 경우 $4\gamma h\nu \ll mc^2$

- In real life, θ in many angle. $h\nu_{s,iso} = \frac{4}{3}\gamma^2 h\nu$ (등방 분포 광자)

Eg. Crab nebula $U \sim 10$ GeV (10^{10} eV) $\gamma = \frac{u}{mc^2} = 2 \times 10^4$. $\gamma^2 = 4 \times 10^8$
↳ 전자에너지

$\nu = 300$ GHz.

$\Rightarrow \nu_s = 1.6 \times 10^{20}$ Hz (γ-ray band)

· Rate of electron E loss → 충돌 횟수, 상호작용 별 전자 에너지 잃기. $= u_{ph}$

$$\left(\frac{du}{dt}\right)_{T_c} \approx - \sigma_T \, n_{ph} \, c \, (h\nu_{s,iso}) = -\frac{4}{3}\sigma_T \, c \, \gamma^2 \, n_{ph} \, h\nu_{av}$$

[photons · $m^{-2} s^{-1}$] "flux"

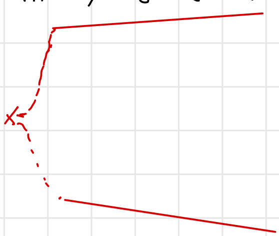
↳ Thompson cross section ($E \ll mc^2$) 에서 가능.

$$\left(\frac{du}{dt}\right)_{IC} = -\frac{4}{3}\sigma_T \, c \, \beta^2 \gamma^2 u_{ph} \; [W] = -2.66 \times 10^{-20} \beta^2 \gamma^2 u_{ph} \quad (J/m^3)$$

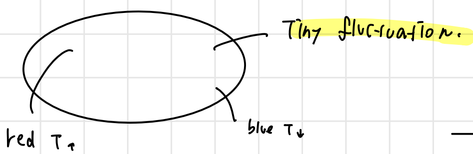
· Volume emissivity

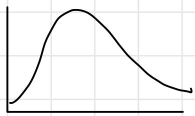
$$j = \frac{4}{3}\sigma_T c \beta^2 \gamma^2 n_e u_{ph} \; [W \cdot m^{-3}] \Leftarrow \int j_\nu d\nu$$

(f) History of Cosmos



CMB : BB ~ 2.73 K (10^{-3} eV)

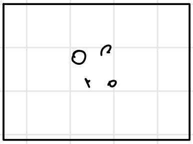




· CMB photon 우리에게 접근. electron E↑ interaction → +SZ 효과

X-ray medium ICM → thermal Bremsstrahlung 으로 확인.

hot ionized plasma → 5 keV ($T \sim 10^8$ K) 상대론 X.



ESA Image

Galaxy cluster

- Optical : galaxy
- X-ray (Bremsstrahlung) : Intracluster medium (ICM)
- Radio / mm (Sunyaev - Ze'ldovich effect) : Inverse Compton Scattering

· Hubble - Lemaitre law

redshift $v = H_0 \cdot r$ → distance ladder (error) ⇒ (X선 data, SZ 효과) 함께!

ok

· Specific intensity (Bremsstrahlung)

$$I_x(\nu_x, T_e) = \frac{C_1}{4\pi} \frac{g(\nu_x, T_e)}{T_e^{1/2}} \exp\left(-\frac{h\nu_x}{k_B T_e}\right) n_e^2 \, (2R)$$

electron density, size

· Observation at X-ray (several V_x share 보다)

→ T_e 을 측정가능 $\frac{\Delta T_r}{T_r} = -2 \frac{k_B T_e}{mc^2} \sigma_T n_e \cdot 2R$

↳ CMB온도

→ $I_x(n_e, R)$

+ Imaging → θ (apparent size)

Combining

X-ray + Radio/mm

⇒ n_e, "$\underline{\underline{R}}$" 구하기 가능

⇒ $r = \frac{R}{\theta}$ → distance. where $H_0 = \frac{v}{r} = \frac{cz}{R/\theta}$ directly observe

derive from observation

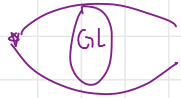
⇒ Combining multi-wavelength observation, physical understanding → H_0 측정가능

5. Gravitational Lensing.

→ Discovery : Sir Edington. (1919 Solar eclipse)

→ Solar system에서 관찰 → 1.75 arc sec. → 이론치와 일치 (정확히는, 뉴턴과의 불일치)

→ 다른 별의 가능성 논의 (아인슈타인, 1936) cf) 비관적)



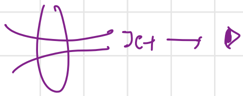
→ SMBH G.L 논의

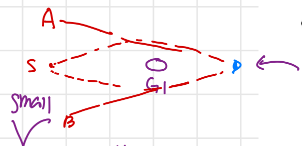
→ Zwicky : Galaxy as a lens (Nabulae as gravitational lens)

→ 1979 : optical / radio 카테고라이싱 중 흥미로운 발견

Same Spectrum

→ 이중 Quasar





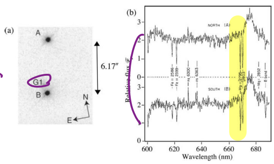


Figure 12.1. "Twin quasar" Q 0957+561. (a) Image taken with the WFPC2 camera on the Hubble Space Telescope. The lensing galaxy G1 is evident as a faint nebulosity. It is embedded in a cluster of galaxies that contribute to the lensing. The ray yielding the lower image passes deeper into the cluster potential. (b) Optical spectra of the two components of Q 0957+561. A broad emission line of Mg II is centered in each spectrum at $\lambda = 675.3$ nm. The rest wavelength of this line is $\lambda_0 = 279.79$ nm, which, from (1), gives a redshift of $z = 1.41$. Five narrow absorption lines of Fe II, Mg II, and Mg I appear at redshift $z = 1.39$ in both spectra. Atmospheric night-sky "ns" emission and B-band absorption are marked. [(a) G. Bernstein *et al.*, *ApJ* **483**, L79 (1997); (b) R. Weymann *et al.*, *ApJ* **233**, L43 (1979)]

spectrum

Radio 영역에서도 robe 동일 발견 ←

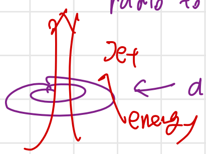
→ 일관성 확인!

z 큼 → 거리↑ → 광도 ⇑

(매우 작고 밝은 천체)

radio to γ-ray

⇒ B.H $\sim 10^8 M_\odot$



스펙트럼 (시간 전광)변화 → Twin Quaisar or...

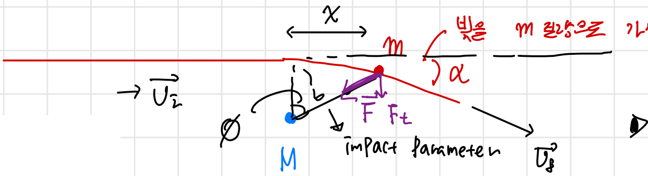
cf) 실제 이중퀘이사 존재 ⇒ 현재까지 ~20개.

G.L !!

→ 5 은하 합체일. → $z=0.39$ 중간 은하 발견. G1.

Gravitational Lensing (매우 작은 굴절각) ← "point mass"

"Newtonian derivation" → 2배 작음.



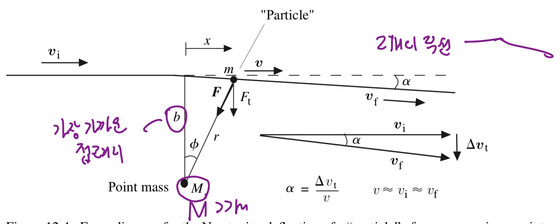
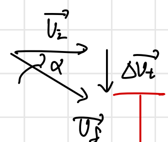


Figure 12.4. Force diagram for the Newtonian deflection of a "particle" of mass m passing a point mass M. The initial and final velocities, v_i and v_f, yield the transverse velocity Δv_t and hence the bending angle α. The deflection is assumed to be very small; the particle motion deviates only slightly from a straight line. The Newtonian scattering angle for a particle at speed c is a factor of two less than that derived from GR for the deflection of a photon.



Transverse Component of Velocity change

$M >> m$, $m v_i$

$$d\vec{v_t} = a_t dt = \frac{GM}{r^2}\cos\phi\, dt$$

$$\vec{F} = -\frac{GMm}{r^2}\hat{r}, \quad F_t = \frac{GMm}{r^2}\cos\phi$$

$$\int_{-\infty}^{\infty} \frac{dx}{(b^2+x^2)^{\frac{3}{2}}} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{b\sec^2\theta\, d\theta}{b^3\sec^3\theta} = \frac{1}{b^2}\cdot 2.$$

$x = b\tan\theta$

For small $\alpha \ll 1$;

$$r \approx \sqrt{b^2+x^2} \qquad \cos\phi = \frac{b}{r} \qquad dt = \frac{dx}{v}$$

$$\therefore v_t = \int dv_t = \int_{-\infty}^{\infty} \frac{GMb}{(b^2+x^2)^{\frac{3}{2}}}\frac{dx}{v} = \underbrace{\frac{2GM}{bv}}_{\frac{2}{b^2}\times\frac{GMb}{v}}$$

$$\longrightarrow \boxed{\alpha \approx \frac{\underbrace{\Delta v_t}_{=v_t}}{v} = \frac{2GM}{bv^2}}$$

$$\xrightarrow{v=c} \boxed{\alpha_{New} \approx \frac{2GM}{bc^2}}$$

... Newtonian light banding angle.

2배차이.

$$\longrightarrow \boxed{\alpha_{GR} = \frac{4GM}{bc^2}}$$

... General relativistic banding angle

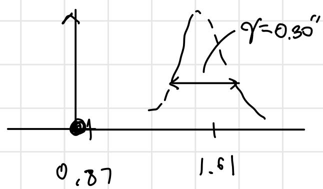
(*) Example : 1919, Eddington, eclipse

$$\frac{1}{3600}\text{ degree} \times \frac{2\pi\text{ rad}}{360\text{ deg}}$$

$b = R_\odot$, $\quad \alpha_{GR} = 8.49\times10^{-6}$ rad $= 1.75''$ (arc sec)

$\alpha_{New} = 0.87''$

$\alpha_{Edd} = 1.61'' \pm 0.30''$ — ok!



$\Rightarrow$ 1974; $\alpha = 1.761'' \pm 0.016''$

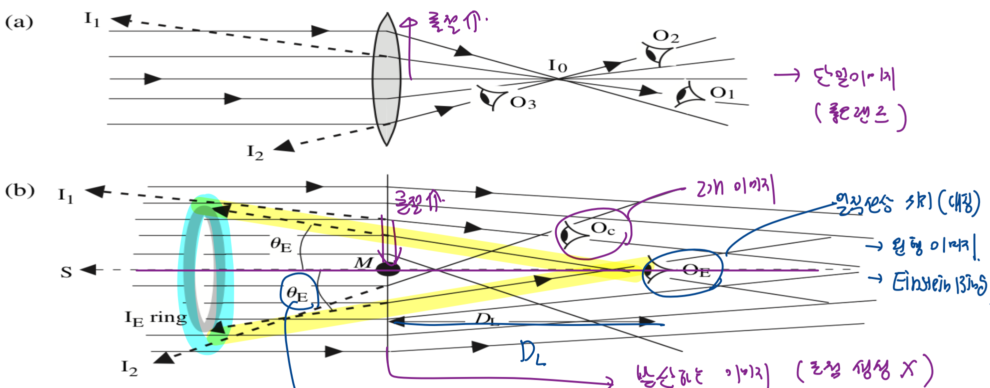


Figure 12.5. Comparison of (a) an ideal convex thin lens and (b) a gravitational point-mass lens, each for incident parallel rays from an infinitely distant source. The bending angle increases with distance from the lens center in the convex case and decreases inversely with distance in the point-mass case. Observers intercept only rays arriving at their local positions, and each such ray acts locally as a bundle of parallel rays focused to a single spot by the eye. Directions to images are indicated with heavy dashed lines.

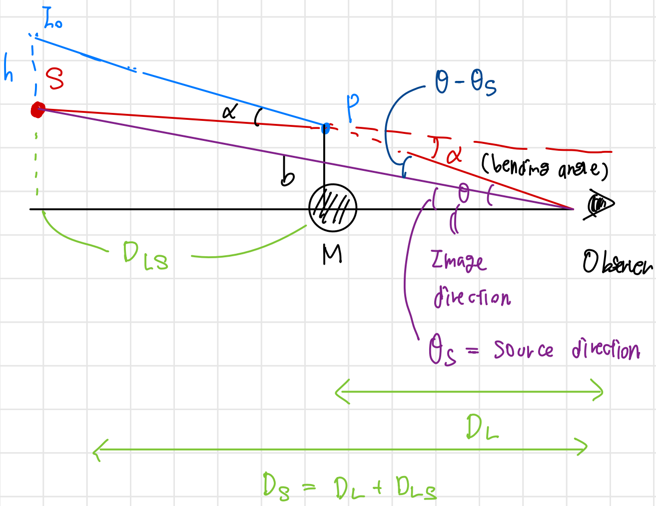
$$\theta_E = \sqrt{\frac{4GM}{D_L c^2}} \quad (\theta_E \ll 1)$$

GL : Image Positions

$$\left.\begin{aligned}\alpha &= \frac{4GM}{bc^2} \\ b &= D_L \theta\end{aligned}\right] \quad \alpha = \frac{4GM}{c^2 D_L \theta} \quad [\text{rad}]$$

D_L → (Spectral) z of the spectral lines

θ → Observation (imaging)



"Ray-trace Equation"

$$\alpha = \frac{h}{D_{LS}} \; ; \quad \theta - \theta_S = \frac{h}{D_S}$$

$$\Rightarrow \alpha = \frac{D_S}{D_{LS}}(\theta - \theta_S)$$

"Lens equation"

$$\boxed{\frac{4GM}{c^2 D_L \theta} = \frac{D_S}{D_{LS}}(\theta - \theta_S)}$$

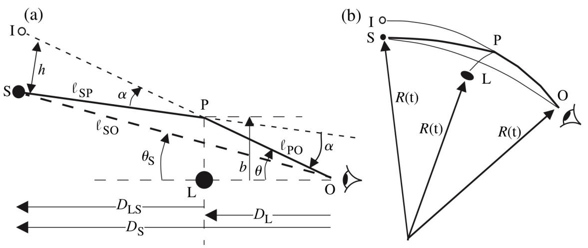


Figure 12.6. (a) Euclidean geometry for deflection of a ray by a galaxy taken to be a point-mass lens. The deflection is approximated as occurring at a single point P at a distance *b* from the lens (b) The direct and bent paths of (a) on an expanding spherical surface, as in a closed universe with positive curvature.

"Einstein ring"

- $\theta_S = 0$.

$$\Rightarrow \theta_E = \sqrt{\frac{4GM}{c^2} \cdot \frac{1}{D}}$$

$$\left(D \equiv \frac{D_L D_S}{D_{LS}} \right)$$

"$\theta^2 - \theta_S \theta - \theta_E^2 = 0$"

$$\rightarrow \theta_{12} = \frac{\theta_S \pm \sqrt{\theta_S^2 + 4\theta_E^2}}{2}$$

$$\Rightarrow \theta_E^2 = \frac{4GM}{c^2} \cdot \frac{D_{LS}}{D_L D_S}$$

$$\rightarrow M = \frac{c^2 \theta_E^2}{4G} D$$

~ P 452.

$$\frac{\theta_E^2}{\theta} = \theta - \theta_S$$

Galaxy

DESI (Dark Energy Spectroscopic Instrument) Arizona

Galaxy → test particle

M87 → Jet structure (global signature even in others)

Galaxy mass & SMBH mass → some correlation. 0.001? 10^{11}

Galaxy classification → 연구중.

CO → weight → cold gas

$$2\sum E_k + \sum E_p = 0 \Rightarrow M = \frac{2\langle v_r^2\rangle_{av}}{G\langle r_{ij}^{-1}\rangle_{av}}$$

$\sim 10^{14} M_\odot$

→ pair!

Composition of Galaxy cluster

Galaxies ~ 1%

DM ~ 90%

IGM/ICM ~ 9%

Stars $M_{visible} \sim 10^{12}$

total $M_{dynamical} \sim 10^{14}$

flow gas $M_{X\text{-}ray} \sim 2\times10^{13}$

2. FIR luminosity of a star forming galaxy

$$L_{1.4\,\mathrm{GHz}} \propto L_{FIR}$$

땅에서 가능 　　　　 space mission 필요.

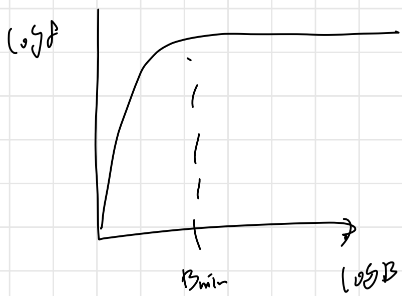
Synchrotron. ⟷ Relativistic electron.

IC radiation

$\log f$

↓

$$\frac{\left(\frac{dU}{dt}\right)_{syn}}{\left(\frac{dU}{dt}\right)_{syn} + \left(\frac{dU}{dt}\right)_{IC}}$$



ULIRGs

$L \sim 10^{11.5} L_\odot$

$R \sim 100\,pc$

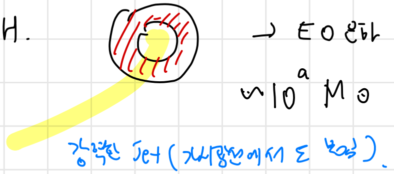
$U_{rad} > U_{ph}$

은하 radio → infrared → optical → ultraviolet → x-ray

cold gas	red star	sun-like	Hot, blue	Ionized.
∃ CO_2	$< M_\odot$	$\sim M_\odot$	$> 1 M_\odot$	very hot

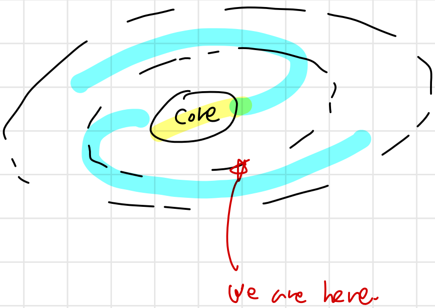
) ∈ Galaxy!

M87 → 거대은하, SMBH.

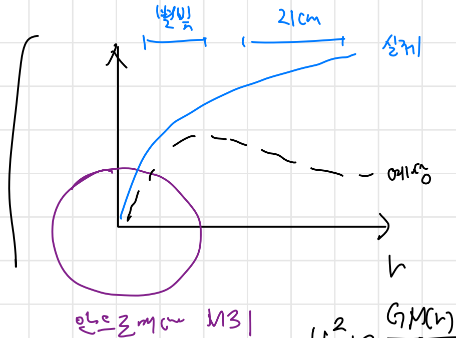


→ E0 은하 (~원)
$\sim 10^9 M_\odot$

→ 구형 대칭, 늙은대형, 매우 밝음.



Missing mass



$v^2 \sim \frac{GM(r)}{r}$

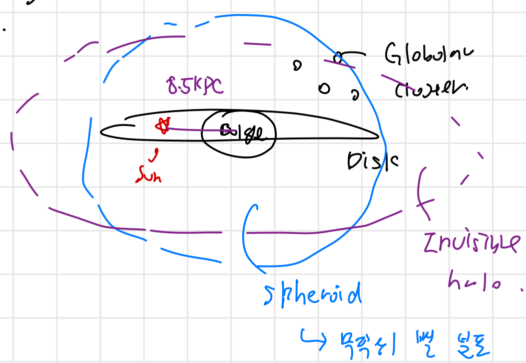
⟨Milky way spiral Galaxy⟩ → 10^{11} [Disk / bulge / halo.

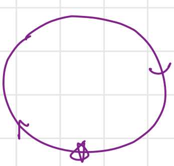
Galaxy
- 우주 기본 단위, 거대 구조물의.
- broad range of property. → 진화 (형성을 반영., 형태 분류 ex) 허블.
- 종류
- $\sim 10^{11}$ 개 우리 관측가능 우주
- $10^8 \sim 10^{12} M_\odot$, $10^7 \sim 10^{11}$ 별.

) −1 ⟨M⟩ = $10 M_\odot$?

Out → Galaxy 관측 (외부 은하)

중성수소(HI) → 21cm . 1100만년 주기
→ 매우 N HI, 관측 가능.

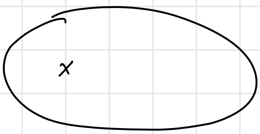




12/10 (화)

은하수 (Milky way) $N \sim 10^{11}$ stars → 10^{11} galaxy in 관측가능우주.

대부분 은하 중심 → SMBH.



→ binary, ellipsoidal

⇒ X - cynus blackhole

exoplanet.

Sagittarius A*

"adaptive optics"

※ DM History